Some Special Cases of Spin-Yaw Lock-In

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A slightly asymmetric missile is a basically symmetric missile with a nonzero pitch moment at zero angle of attack. In flight, this moment causes a trim angle that rotates with the missile and has a maximum when the spin is near resonance with the missile's natural pitch frequency. This paper considers a roll moment that can be induced by this trim angle and can cause resonant lock-in spin. Simple expressions for this induced roll moment are given, and the conditions for equilibrium spins and their stability are derived. The special case of the induced roll moment caused by a radial center-of-mass offset is considered, and a number of different types of possible equilibrium spin combinations are shown. The possibility of resonant lock-in spin in the opposite sense to the expected steady-state spin is indicated. Furthermore, it is shown that there are induced roll moments for which the design steady-state spin will not occur under any launch conditions. If several stable equilibrium spins are possible, the one that occurs in flight can be determined by the orientation of the initial pitch angular velocity. Finally, the form of an induced pitch moment is given, and its possible effect on the angular motion is discussed.

Nomenclature

C_D = drag force coefficient, = drag force/$\frac{1}{2}\rho V^2 S$
C_{L_α} = lift force coefficient, = $C_{N_\alpha} - C_D$
C_l = $M_x/\frac{1}{2}\rho V^2 Sl$, where M_x is the axial aerodynamic moment
C_{l_p} = spin-damping moment coefficient, Eq. (12)
C_{l_δ} = spin-producing moment coefficient due to canted fins, Eq. (12)
C_{l_θ} = spin moment coefficient induced by angle of attack, Eq. (12)
$C_{M_{p\alpha}}$ = Magnus moment coefficient, Eq. (2)
$C_{M_q} + C_{M_{\dot\alpha}}$ = damping moment coefficient sum, Eq. (2)
C_{M_0} = asymmetry moment coefficient, Eq. (2)
C_{M_α} = static moment coefficient, Eq. (2)
C_{M_θ} = induced pitch moment coefficient, Eq. (21)
$C_{\tilde m}, C_{\tilde n}$ = $(M_{\tilde y}, M_{\tilde z})/\frac{1}{2}\rho V^2 Sl$, where $M_{\tilde Y}, M_{\tilde z}$ are the transverse aerodynamic moments in the aeroballistic system
C_{N_0} = asymmetry force coefficient, Eq. (1)
C_{N_α} = normal force coefficient, Eq. (1)
$C_{\tilde y}, C_{\tilde z}$ = $(F_{\tilde y}, F_{\tilde z})/\frac{1}{2}\rho V^2 S$, where $F_{\tilde y}, F_{\tilde z}$ are the transverse aerodynamic forces in the aeroballistic system
F_N = normal force due to radially offset center of mass
f_1 = $iG(\zeta^n - \bar\zeta^n)$
f_2 = $\phi_e - \phi_s$
G = $\hat K_\theta \delta_{TR}^n$
H = $(\rho Sl/2m)[C_{L_\alpha} - C_D - k_t^{-2}(C_{M_q} + C_{M_{\dot\alpha}})]$
h = $(\hat H - \sigma \hat T)/(1 - \sigma)$
I_x = axial moment of inertia
I_t = transverse moment of inertia
K_p = $-(\rho Sl^3/2I_x)[C_{l_p} + k_a^2 C_D]$, Eq. (18)
K_δ = $(\rho Sl^3/2I_x)\delta_f C_{l_\delta}$, Eq. (18)
K_θ = $(b_{10}/2)[C_{l_p} + k_a^2 C_D]^{-1}$, Eq. (18)
k_a^2 = I_x/ml^2
k_t^2 = I_t/ml^2
l = reference length (diameter)
M = $(\rho Sl^3/2I_t)C_{M_\alpha}$
M_A = $-(\rho Sl^3/2I_t)C_{M_0}$
$M_{x\theta}$ = roll moment due to radially offset center of mass, Eq. (31)
m = mass
N = type designator: normal resonant lock-in
n = symmetry number; symmetry angle is $2\pi/n$ rad
R = type designator: reverse resonant lock-in
$\hat r_c$ = radial offset of the center of mass, calibers
S = reference area $\pi l^2/4$; type designator: steady-state spin
s = $\int_0^t (V/l)\mathrm{d}t$
T = $(\rho Sl/2m)[C_{L_\alpha} + k_a^{-2} C_{M_{p\alpha}}]$
t = time
V = magnitude of the velocity
α, β = angles of attack and sideslip in the missile-fixed system, respectively
$\tilde\alpha, \tilde\beta$ = angles of attack and sideslip in the aeroballistic system, respectively
δ = absolute value of ξ and $\tilde\xi$
δ_f = differential fin cant angle
δ_t = absolute value of ξ_T
δ_{TR} = $\delta_{T0}/|h|$
δ_{T0} = $M_A/M = -C_{M_0}/C_{M_\alpha}$
ζ = ξ/δ_{TR}
$\dot\zeta_0$ = complex angular velocity at $\tau = 0$
θ = orientation angle of ξ, Eq. (9)
$\tilde\theta$ = orientation angle of $\tilde\xi$, Eq. (10)
θ^* = orientation angle of $\dot\zeta_0$
λ = constant $\dot\eta_j \eta_j$, Eq. (29)
ξ = complex angle of attack in the missile-fixed system, $= \beta + i\alpha = \delta e^{i\theta}$
$\tilde\xi$ = complex angle of attack in the aeroballistic system, $= \tilde\beta + i\tilde\alpha = \delta e^{i\tilde\theta}$
ξ_T = constant trim angle value of ξ, Eq. (8)
ρ = air density
σ = I_x/I_t
τ = $[-M/(1-\sigma)]^{1/2} s$
ϕ = roll angle
ϕ_M = asymmetry moment orientation angle, Eq. (2)
ϕ_N = asymmetry force orientation angle, Eq. (1)
ϕ_s' = steady-state spin, rad/calibers = K_δ/K_p
ϕ_T = $\tan^{-1}[-\phi h/(1 - \phi^2)]$, Eq. (8)

Superscripts

$(\bar{\ })$ = complex conjugate
$(\hat{\ })$ = $[-(1-\sigma)/M]^{1/2}(\ )$
$(\dot{\ })$ = $\mathrm{d}(\ )/\mathrm{d}\tau$
$(\ )'$ = $\mathrm{d}(\ )/\mathrm{d}s$

Subscripts

e = equilibrium value
s = steady-state value

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Introduction

IN 1953, Nicolaides introduced the concept of spin-yaw resonance of missiles having slight configurational asymmetries.[1] These asymmetries cause a constant-amplitude pitch moment, which rotates with the missile. In Ref. 1, Nicolaides showed the possibility of large trim angles when the spin rate was near the natural pitch frequency of the missile. Subsequent authors considered various aspects of the pitching motion of the missile when its spin was near resonance.[2-5] The combination of symmetric nonlinear aerodynamic moments and a trim moment produced by a configuration asymmetry has been studied extensively for nonresonant spin and the possible existence of subharmonic response. A variety of limit motions also have been demonstrated.[6-10]

The very important question of the existence of a spin moment that forces the spin to its resonance value was first answered by Nicolaides[2] when he introduced an induced roll moment, which was a function of the total angle of attack and the roll angle between a particular fin and the plane of the total angle of attack. (This "induced" roll moment is induced by the nonrotationally symmetric flowfield over a finned missile at angle of attack.) Pepitone[9] studied the effect on the missile's angular motion of this induced roll in conjunction with an induced lift force and an induced pitch moment. Reference 9 did not, however, include a trim moment in its analysis and, thus, did not consider the possibility of spin-yaw lock-in.

Glover[11] considered the effect of mass and aerodynamic asymmetries on spin. He showed that a laterally offset center-of-mass (c.m.) location introduces an induced roll moment, which is a function of two angles: 1) the total angle of attack and 2) the angle between the angle-of-attack plane and the plane containing the c.m. and axis of symmetry. This analysis has been extended to sounding rockets and re-entry vehicles.[12,13]

In this paper, we will consider the variety of spin lock-ins that can occur for a slightly asymmetric missile whose rolling motion is controlled by the usual linear roll moments and a generalized induced roll moment. Three different types of lock-in are shown, and conditions on the induced roll moment are given.

Slightly Asymmetric Missile

A missile whose linear aerodynamic forces and moments have the same symmetry as those for a body of revolution will be called "basically symmetric." If this basic symmetry is disturbed so that its normal force and static moment are not zero at zero angles of attack and sideslip, the missile is a "slightly asymmetric" missile. (A slightly asymmetric missile has equal zero-spin pitch and yaw frequencies. If these frequencies differ slightly, the missile is called an "almost symmetric" missile.[14]) This disturbance can be caused by small cant angles of fin surfaces and induces a trim pitch angle that rotates with the missile and is a function of the spin rate. When the spin rate is near the fast precession rate, a maximum value of the trim angle occurs. This event is called "spin-pitch resonance."

Aeroballistic axes pitch and yaw with the missile but have zero roll rate. The linear aerodynamic force and moment for a slightly asymmetric missile can be written in these coordinates as

$$C_{\tilde{y}} + iC_{\tilde{z}} = -C_{N_\alpha}\tilde{\xi} - C_{N_0}\exp[i(\phi + \phi_N)] \quad (1)$$

$$C_{\tilde{m}} + iC_{\tilde{n}} = [\phi' C_{M_{p\alpha}} - iC_{M_\alpha}]\tilde{\xi} - i(C_{M_q} + C_{M_{\dot\alpha}})\tilde{\xi}'$$
$$- iC_{M_0}\exp[i(\phi + \phi_M)] \quad (2)$$

where C_{N_0} and C_{M_0} are nonnegative.

This force and moment can be inserted into the usual differential equations for the pitching and yawing motion[15]:

$$\tilde{\xi}'' + (H - i\sigma\phi')\,\tilde{\xi}' - (M + i\sigma\phi' T)\tilde{\xi} =$$
$$- M_A \exp[i(\phi + \phi_M)] \quad (3)$$

where

$$M_A = -\frac{\rho S l^3}{2I_t}\{C_{M_0} - ik_t^2(1-\sigma)\phi' C_{N_0}\exp[i(\phi_N - \phi_M)]\}$$

At resonance, ϕ' is of the order 10^{-2}; therefore, the C_{N_0} term in M_A can and will be neglected. For zero spin, the steady-state motion can be described by a constant trim angle:

$$\tilde{\xi} = \delta_{T0}\, e^{i\phi_M} \quad (4)$$

The independent variable in Eq. (3) is dimensionless time, s. For spin-yaw resonance, it is convenient to use a second dimensionless time, τ, which is defined as

$$\tau = [-M/(1-\sigma)]^{1/2} s \quad (5)$$

Equation (3) now becomes

$$\ddot{\tilde{\xi}} + (\hat{H} - i\sigma\dot\phi)\dot{\tilde{\xi}} + (1 - \sigma - i\sigma\dot\phi\,\hat{T})\,\tilde{\xi}$$
$$= \delta_{T0}\,(1-\sigma)\exp[i(\phi + \phi_M)] \quad (6)$$

where

$$\hat{H} = H[-(1-\sigma)/M]^{1/2}$$
$$\hat{T} = T[-(1-\sigma)/M]^{1/2}$$

For constant spin, we assume that the steady-state response to the aerodynamic trim has the form

$$\tilde{\xi} = \xi e^{i\phi} \quad (7)$$

where

$$\xi = \xi_T \text{ const}$$

A direct substitution of Eq. (7) into Eq. (6) yields

$$\xi_T = \frac{\delta_{T0}\exp(i\phi_M)}{1 - \dot\phi^2 + i\dot\phi h} = \delta_T \exp[i(\phi_T + \phi_M)] \quad (8)$$

where

$$h = (1-\sigma)^{-1}\,[\hat{H} - \sigma\hat{T}]$$

and dynamic stability near resonance requires that $h > 0$.[15]

The simple form of Eq. (8) results from the use of τ as the independent variable. According to Eq. (8), resonance occurs at $|\dot\phi| = 1$; the resonance value of the trim angle magnitude is $\delta_{TR} = |h|^{-1}\delta_{T0}$. The magnitude of the trim angle grows from δ_{T0} to a maximum, which is approximately δ_{TR}, and then decays to zero. The phase angle varies from zero at zero spin to -90 deg at resonance and then to -180 deg for infinite spin.

Roll Equation

The roll moment for a slightly asymmetric missile usually has two components[16]: a constant spin-producing moment caused by differential cant of the fin surfaces, δ_f, and a spin-damping moment proportional to the spin rate. If the missile is not a body of revolution, a roll moment can be induced by an angle of attack and varies with θ, the angular orientation of the angle-of-attack plane with respect to the missile.

In the usual missile-fixed axes, the complex angle of attack has the form

$$\xi = \beta + i\alpha = \delta e^{i\theta} \tag{9}$$

Since the aeroballistic axes differ from these axes by the roll angle ϕ,

$$\tilde{\xi} = \delta e^{i\tilde{\theta}} = \xi e^{i\phi} \tag{10}$$

Therefore

$$\tilde{\theta} = \theta + \phi \tag{11}$$

The complete roll moment including the angle-of-attack-induced component can be written as

$$C_l = C_{l_\delta}\delta_f + C_{l_p}\phi' + C_{l_\theta}(\theta,\delta) \tag{12}$$

The Y axis in the missile-fixed coordinates is usually taken to be in a plane of mirror symmetry of the basically symmetric missile. If this is the case, the induced roll moment coefficient is an odd function[17] of θ.

$$C_{l_\theta}(\theta,\delta) = -C_{l_\theta}(-\theta,\delta) \tag{13}$$

Therefore, it can be expanded as a Fourier sine series in θ. A rotationally symmetric missile with n similar fins has a symmetry angle of $2\pi/n$; thus, the induced roll moment should have this fundamental wavelength.

$$C_{l_\theta} = \sum_{k=1}^{\infty} a_k \sin nk\theta \tag{14}$$

where

$$a_k = a_k(\delta) \qquad \text{and} \qquad a_k(0) = 0$$

In terms of the complex angle of attack,

$$\sin nk\theta = \frac{\xi^{nk} - \bar{\xi}^{nk}}{2i\delta^{nk}} \tag{15}$$

If we make the mathematically attractive assumption[17] that the roll moment is an analytic function of α and β, the $a_k(\delta)$ function can be expressed as a special power series in δ.

$$a_k = \delta^{nk} \sum_{l=0} b_{kl}\delta^l \tag{16}$$

Equations (14) and (16) give a double series expression for the induced roll moment coefficient. For the calculations herein, we will approximate the induced roll moment by the first term in this series.

$$C_{l_\theta} = b_{10}\,\delta^n \sin n\theta$$

$$= -ib_{10}\,(\xi^n - \bar{\xi}^n)/2 \tag{17}$$

As a further convenience, we will select the orientation of the Y axis in the plane of mirror symmetry so that b_{10} is positive. The resulting roll equation for this roll moment is as follows[15]:

$$\phi'' + K_p[\phi' - \phi'_s - iK_\theta\,(\xi^n - \bar{\xi}^n)] = 0 \tag{18}$$

For most finned missiles, K_p is positive. In the absence of an induced roll moment, Eq. (18) predicts a stable steady-state spin of ϕ'_s. This design steady-state spin is set by the designer by varying the fin differential deflection angle, δ_f.

The independent variable in Eq. (18) can be easily changed from s to τ, and ξ can be scaled by its value at resonance.

$$\ddot{\phi} + \hat{K}_p[\dot{\phi} - \dot{\phi}_s - iG(\zeta^n - \bar{\zeta}^n)] = 0 \tag{19}$$

where

$$\hat{K}_p = K_p[-(1-\sigma)/M]^{1/2}$$

$$G = K_\theta[-(1-\sigma)/M]^{1/2}\,\delta^n_{\mathrm{TR}}$$

$$\zeta = \xi\,\delta^{-1}_{\mathrm{TR}}$$

For linear aerodynamic force and moment, Eq. (6) could be divided by δ_{TR} to yield a differential equation for the scaled angle of attack, ζ. Unfortunately, if we allow an induced roll moment to be present, the normal force and pitch moment can have nonlinear components dependent on roll-pitch plane orientation,[9,17]

$$(C_{\tilde{y}} + iC_{\tilde{z}})_\theta = -C_{N_\theta}(\theta,\delta)\,\xi e^{i\phi} \tag{20}$$

$$(C_{\tilde{m}} + iC_{\tilde{n}})_\theta = -i\,C_{M_\theta}(\theta,\delta)\,\xi e^{i\phi} \tag{21}$$

For simplicity, we will neglect these components and retain the linear form of Eq. (6). Should these induced force and moment terms be too large to neglect, they can cause the "catastrophic yaw" described by Nicolaides.[2] The differential equation for the scaled complex angle of attack, ζ, can now be obtained from Eq. (6):

$$\ddot{\zeta} + [\hat{H} + i(2-\sigma)\dot{\phi}]\,\dot{\zeta} + (1-\sigma)[1 - \dot{\phi}^2 + i\dot{\phi}h + i\ddot{\phi}]\,\zeta$$

$$= (1-\sigma)\,|h|\,e^{i\phi_M} \tag{22}$$

Lock-in occurs when Eqs. (19) and (22) have a constant steady-state equilibrium solution

$$\dot{\phi}_e - \dot{\phi}_s = iG\,(\zeta^n_e - \bar{\zeta}^n_e) \tag{23}$$

$$\zeta_e = \frac{|h|\;e^{i\phi_M}}{1 - \dot{\phi}^2_e + i\dot{\phi}_e h} \tag{24}$$

ζ_e has a maximum amplitude of unity. Thus, if G is small compared to $\dot{\phi}_s$, the induced roll moment has very little effect on equilibrium spin, and the usual steady-state spin occurs. If, however, this is not the case, the induced roll moment can have a large contribution for $\dot{\phi}_e \cong \pm 1$ and the roll rate can be locked in at resonant spin ($\dot{\phi}^2 = 1$). At resonance, θ is $\phi_M - (\pi/2)$. As we shall see, lock-in can occur both for the normal case of resonant spin in the same sense as the expected steady-state spin and for the reverse case of resonant spin with the opposite sense.

Lock-In Stability

A number of solutions to Eqs. (23) and (24) can exist. These may be found graphically by plotting the curve

$$y = f_1(\dot{\phi}_e) = iG(\zeta^n - \bar{\zeta}^n) \tag{25}$$

and finding its intersection with the line

$$y = f_2(\dot{\phi}_e) = \dot{\phi}_e - \dot{\phi}_s \tag{26}$$

Some of these equilibrium points, however, may be unstable and have no engineering significance.

We will assume a small perturbation of an equilibrium solution.

$$\dot{\phi} = \dot{\phi}_e + \eta_1 \tag{27}$$

$$\zeta = \zeta_e + \eta_2 + i\eta_3 \tag{28}$$

where $\eta_j = \eta_j(\tau)$.

Substitution of Eqs. (27) and (28) into Eqs. (19) and (22) yields a fifth-order differential system in η_j. Next, we assume

Table 1 Stability matrix elements, a_{jk}

$$a_{11} = \lambda + \hat{K}_p$$

$$a_{12} = in\, \hat{K}_p\, G(\zeta_e^{n-1} - \bar{\zeta}_e^{n-1})$$

$$a_{13} = n\, \hat{K}_p\, G(\zeta_e^{n-1} + \bar{\zeta}_e^{n-1})$$

$$a_{21} = -R\{\zeta_e[(2\dot{\phi}_e - ih)(1-\sigma) + i\lambda]\}$$

$$a_{22} = \lambda^2 + \hat{H}\lambda + (1-\sigma)(1-\dot{\phi}_e^2)$$

$$a_{23} = -\dot{\phi}_e[(2-\sigma)\lambda + (1-\sigma)\, h]$$

$$a_{31} = -I\{\zeta_e[(2\dot{\phi}_e - ih)(1-\sigma) - i\lambda]\}$$

$$a_{32} = \dot{\phi}_e[(2-\sigma)\lambda + (1-\sigma)\, h]$$

$$a_{33} = \lambda^2 + \hat{H}\lambda + (1-\sigma)(1-\dot{\phi}_e^2)$$

a coupled exponential solution for these perturbation functions:

$$\eta_j = \eta_{j0}\, e^{\lambda\tau} \tag{29}$$

where η_{j0} is constant.

Direct substitution gives three equations, one for each η_{j0}.

$$\sum_{j=1}^{3} a_{jk}\, \eta_{j0} = 0 \qquad k = 1,2,3 \tag{30}$$

where a_{jk} is given in Table 1.

The determinant of the system [Eq. (30)] must be zero for a nontrivial solution. This condition reduces to a fifth-order polynomial equation in λ. If all of the roots have negative real parts, the equilibrium is stable.

It should be noted that the location of the equilibrium points is determined by the parameters n, $\dot{\phi}_s$, G, ϕ_M, and h. The stability of these equilibrium point requires values of $\hat{H}$, $\hat{K}_p$, and σ in addition to these five basic parameters.

Center-of-Mass Offset

When n is 3 or greater, a missile has trigonal or greater rotational symmetry, and its linear force and moment coefficients have the same symmetry as a body of revolution.[17] Di-gonal rotational symmetry ($n = 2$) would occur, for example, when a four-fin missile has pairs of fins with unequal areas. The theory for almost symmetric missiles would apply to the motion of di-gonal missiles. Although $n = 1$ implies no rotational symmetry, a symmetric missile with a laterally offset c.m. will have a roll moment term of the form given by Eq. (17) for $n = 1$.

We will assume the center of mass to be radially offset by a distance of $l\hat{r}_c$ in the plane $\phi = 0$. The normal force can then exert a roll moment with a lever arm of $l\hat{r}_c \sin\theta$.

$$M_{x\theta} = F_N\, l\hat{r}_c \sin\theta = -\rho Sl\, V^2(i/4)\, \hat{r}_c\, C_{N_\alpha}(\xi - \bar{\xi}) \tag{31}$$

If Eq. (31) is compared with Eq. (17), we see that the radially offset center of mass produces an induced roll moment with $n = 1$, $b_{10} = \hat{r}_c C_{N_\alpha}$.

The offset center of mass can also produce a trim pitch moment since the drag force now has the linear arm $l\hat{r}_c$.

$$(C_{\tilde{m}} + iC_{\tilde{n}})_0 = -i\, \hat{r}_c C_D\, e^{i\phi} \tag{32}$$

Therefore,

$$\phi_M = 0, \qquad C_{M_o} = \hat{r}_c\, C_D \tag{33}$$

Discussion

According to Eqs. (23) and (24), the induced roll moment has maximum amplitudes near resonance. For even values of n, and $n\phi_M = \pm\pi/2, \pm 3\pi/2, \ldots$, $|f_1|$ has an absolute maximum of $2|G|$ at $\dot{\phi}_e = \pm 1$. For odd values of n, the same absolute maximum occurs at $n\phi_M = \pm 0, \pm\pi, \ldots$. Thus, resonant lock-in is possible only for $|\dot{\phi}_s| < 2G$. In actuality, occurrence of lock-in depends on the specific values of (h, $\dot{\phi}_s$, G, ϕ_M), whereas the stability of the lock-in depends on ($\hat{H}$, $\hat{K}_p$, σ). Throughout this section, $h = \hat{H} = \hat{K}_p = \sigma = 0.1$, and we will consider only different values of $\dot{\phi}$, G, and ϕ_M.

In order to consider lock-in in more detail, the remainder of this discussion will be limited to the case of $n = 1$. For this case of an offset center of mass and $\phi_M = \pi$, f_1 has a minimum of $-2G$ at $\dot{\phi}_e = -1$ and maximum of $2G$ at $\dot{\phi}_e = 1$. A simple analysis further shows that for $\phi_M = \pi/2$, f_1 varies from a minimum near $-G$ at $\dot{\phi}_e = -1 - h$ to a maximum near G at $\dot{\phi}_e = -1 + h$ and similarly varies from a maximum near G at

Table 2 Illustrated examples of various lock-in cases

Case	$\dot{\phi}_s$	G	ϕ_M, deg	Stable $\dot{\phi}_e$	Type
1	3.0	5.0	90	1.01, 2.86	*SN*
2	3.0	5.0	270	−0.98, 3.11	*SR*
3	0.5	1.0	180	−1.03, 0.52, 1.08	*SNR*
4	0.5	0.5	90	0.69, 0.97	*SN*
5	0.5	2.0	270	−0.98, 0.10	*SR*
6	0.5	1.0	90	0.99	*N*
7	0.5	5.0	270	−0.99	*R*

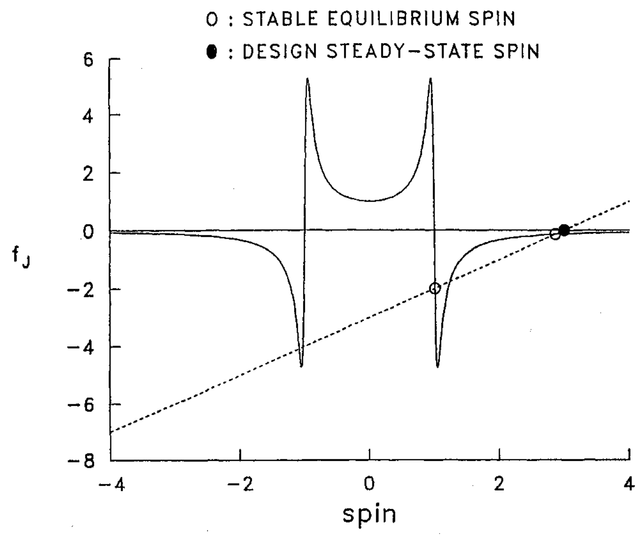

Fig. 1 f_j vs $\dot{\phi}$ **for case 1. Design steady-state spin and normal lock-in.**

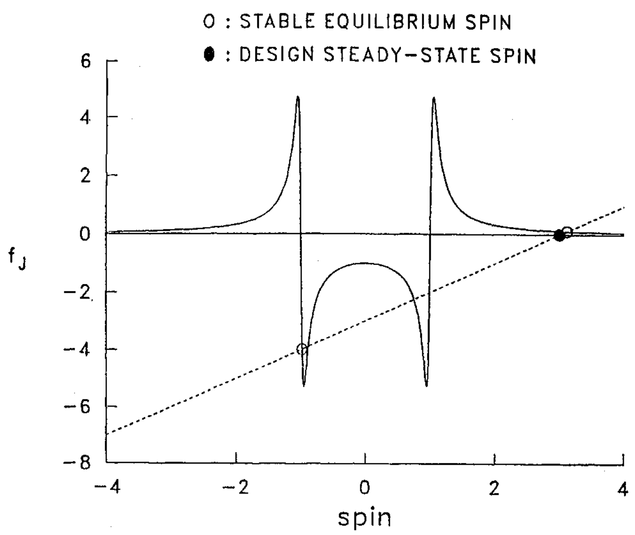

Fig. 2 f_j vs $\dot{\phi}$ **for case 2. Design steady-state spin and reverse lock-in.**

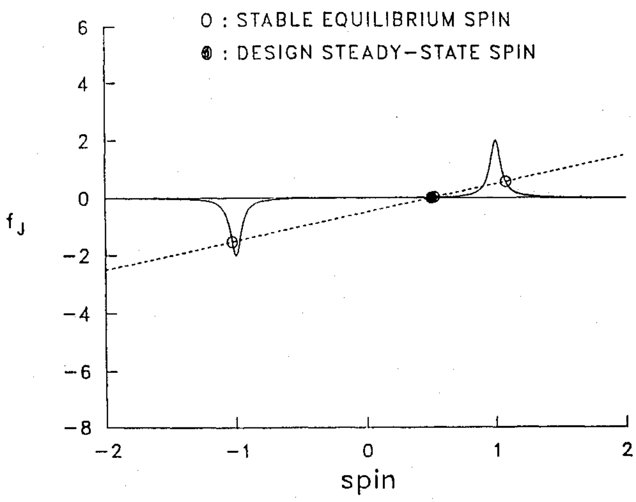

Fig. 3 f_j vs ϕ for case 3. Design steady-state spin, normal lock-in, and reverse lock-in.

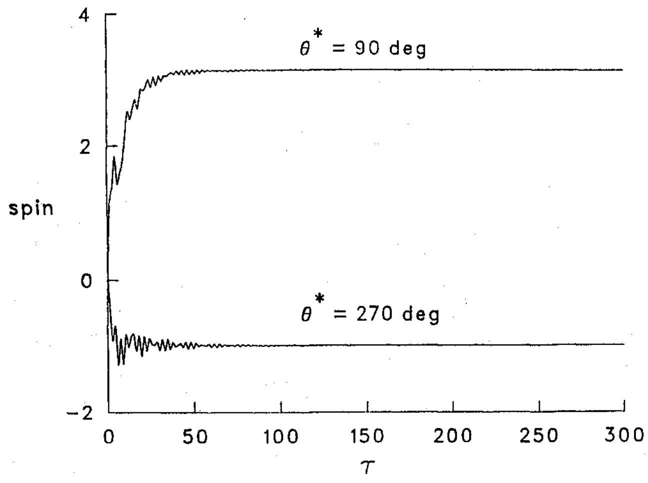

Fig. 5 $\dot{\phi}$ vs τ for case 2 with $|\dot{\zeta}| = 1.0$. Design steady-state spin occurs for $\theta^* = 90$ deg, whereas reverse lock-in occurs for $\theta^* = 270$ deg.

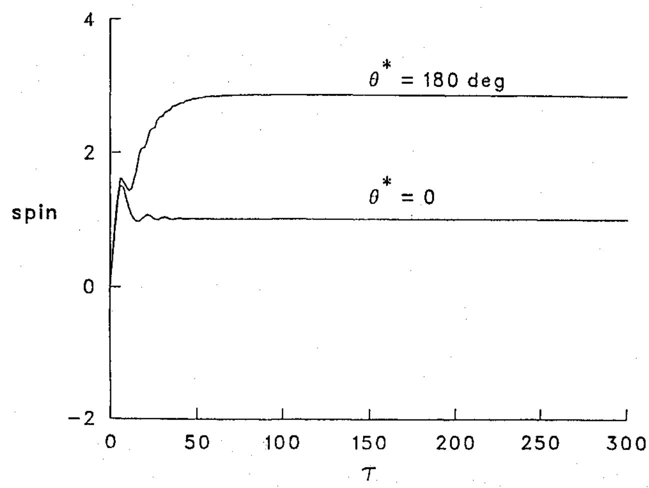

Fig. 4 $\dot{\phi}$ vs τ for case 1 with $|\dot{\zeta}| = 0.1$. Design steady-state spin occurs for $\theta^* = 180$ deg, whereas normal lock-in occurs for $\theta^* = 0$ deg.

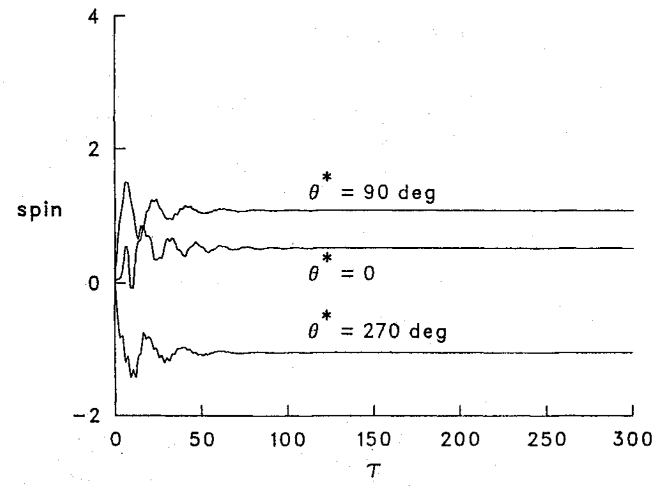

Fig. 6 $\dot{\phi}$ vs τ for case 3 with $|\dot{\zeta}| = 3$. Design steady-state spin occurs for $\theta^* = 0$, normal lock-in for $\theta^* = 90$ deg, and reverse lock-in for $\theta^* = 270$ deg.

$\dot{\phi}_e = 1 - h$ to a minimum near $-G$ at $\dot{\phi}_e = 1 + h$. Finally, $f_1(\theta) = -f_1(\theta + \pi)$. In Figs. 1 and 2, $f_1(\dot{\phi}_e)$ is plotted for $\phi_M = \pi/2$, $3\pi/2$, respectively, for $G = 5$. The dotted lines in these figures are $f_2(\dot{\phi}_e)$ for $\dot{\phi}_s = 3$. The parameters for these figures are the first two entries in Table 2.

In both figures, there are five equilibrium points. In Fig. 1, the slightly modified steady-state spin, $\dot{\phi}_e = 2.68$, and the normal resonance, $\dot{\phi}_e = 1$, are the only stable equilibrium points. This is the normal lock-in- model considered by engineers. If spin starts near zero, designers normally hope that it accelerates through resonance fast enough to avoid capture and reaches steady-state spin with only a slight stimulus to its pitching motion caused by passage through resonance.

In Fig. 2, the two stable equilibrium points are the steady-state spin of 3.11 and the reverse resonance spin of -0.98. This resonant lock-in spin, in the opposite sense to the design spin, is an unexpected result. We will denote normal resonant lock-in spin by *N*, steady-state spin by *S*, and reverse resonant lock-in spin by *R*. Then the two stable equilibrium spins of Fig. 1 could be identified by *SN* and the two stable equilibrium spins of Fig. 2 by *SR*.

If we now consider $\dot{\phi}_s$ to be less than unity, even more remarkable possibilities appear. Five examples are given in Table 2. Case 3 in Table 2 is particularly interesting; the corresponding equilibrium spin determination is shown in Fig. 3. Since $\phi_M = 180$ deg, f_1 has only one maximum and one minimum. There are, however, three stable equilibrium spins; this case is denoted by *SNR*. The next two entries in the table are examples of types *SN* and *SR* for $|\dot{\phi}_s| < 1$ and are quite similar to the first two entries. The final two entries represent new types, *N* and *R*, for which no stable steady-state spin exists. Thus, the induced roll moment can completely overpower the design steady-state spin.

For the cases of two or three stable equilibrium spins, the equilibrium that occurs in flight is determined by initial conditions. Equations (19) and (22) form a fifth-order differential system. The necessary five initial conditions are the initial spin rate $\dot{\phi}_0$, the initial complex angle ζ_0, and the initial complex angular velocity. For simplicity, we will let $\dot{\phi}_0 = \zeta_0 = 0$ and consider only the magnitude and orientation of $\dot{\zeta}_0$.

$$\dot{\zeta}_0 = |\dot{\zeta}_0| \, e^{i\theta^*} \tag{34}$$

For case 1, which was a type *SN* with $\dot{\phi}_s = 3$, $|\dot{\zeta}_0|$ was set at 0.1 and θ^* was varied. As can be seen from Fig. 4, steady-state spin occurs for $\theta^* = 180$ deg and normal lock-in spin for $\theta^* = 0$ deg. For case 2, which was type *SR* with $\dot{\phi}_s = 3$, $|\dot{\zeta}_0|$ was set at 1, steady-state spin occurred for $\theta^* = 90$ deg, and reverse lock-in spin for $\theta^* = 270$ deg (Fig. 5). Finally, for case 3, which was type *SNR* with $\dot{\phi}_e = 0.5$, $|\dot{\zeta}_0|$ was 3, steady-state spin occurred for $\theta^* = 0$, normal lock-in spin for $\theta_b = 90$ deg, and reverse lock-in spin for $\theta^* = 270$ deg (Fig. 6). Therefore, the determination of which equilibrium spin occurs in flight can be made by the orientation of the initial angular velocity.

Conclusions

1) A roll moment can be induced by the pitching and yawing motion of the missile. Simple expressions for this moment have been given for aerodynamically symmetric missiles and for missiles with mass asymmetries.

2) These induced-roll moments can cause the rolling motion of slightly asymmetric missiles to have a variety of steady-state values. Examples of normal resonant lock-in spins in the directions of spin and reverse resonant lock-in spin in the opposite direction are shown.

3) For multiple stable equilibrium spins, the orientation of the initial pitch angular velocity can determine which one occurs in a given flight.

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